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Put $e = \frac{1}{e}$ and $v = \left(\frac{1-e_1^2 v_1^2}{1-v_1^2}\right)^{\frac{1}{2}}$, then by substitution,

$$dH_x = \frac{a(1-e_1^2)dv_1}{(1-v_1^2)^{\frac{3}{2}}(1-e_1^2 v_1^2)^{\frac{1}{2}}}. \quad (2)$$

Let $A = v_1 \left(\frac{1-e_1^2 v_1^2}{1-v_1^2}\right)^{\frac{1}{2}}$, then

$$dH_x = adA - a \left(\frac{1-e_1^2 v_1^2}{1-v_1^2}\right)^{\frac{1}{2}} dv_1 + \frac{a(1-e_1^2)dv_1}{(1-v_1^2)^{\frac{3}{2}}(1-e_1^2 v_1^2)^{\frac{1}{2}}};$$

$$\therefore H_x = aA - aE(e_1, v_1) + \int \frac{a(1-e_1^2)dv_1}{(1-v_1^2)^{\frac{3}{2}}(1-e_1^2 v_1^2)^{\frac{1}{2}}}, \quad (3)$$

where $E(e_1, v_1)$ denotes an elliptic arc, semimajor axis unity, eccentricity e_1 and abscissa v_1 .

Let $e_1 = \frac{1-\sqrt{1-e_2^2}}{1+\sqrt{1-e_2^2}}$ and $v_1 = \frac{2v_2}{1-e_2^2} \left(\frac{1-v_2^2}{1-e_2^2 v_2^2}\right)^{\frac{1}{2}}$, and we get

$$\begin{aligned} \frac{a(1-e_1^2)dv_1}{(1-v_1^2)^{\frac{3}{2}}(1-e_1^2 v_1^2)^{\frac{1}{2}}} &= 2ae_1 dv_1 + 2a \left(\frac{1-e_1^2 v_1^2}{1-v_1^2}\right)^{\frac{1}{2}} dv_1 \\ &\quad - 2a(1+e_1) \left(\frac{1-e_2^2 v_2^2}{1-v_2^2}\right)^{\frac{1}{2}} dv_2. \end{aligned} \quad (4)$$

$$\therefore H_x = aA + 2ae_1 v_1 + aE(e_1, v_1) - 2a(1+e_1)E(e_2, v_2). \quad (5)$$

See *Wright's Solutions of the Cambridge Problems*, vol. ii, pp. 148—9.

A PROBLEM IN LEAST SQUARES.

BY R. J. ADCOCK, MONMOUTH, ILL.

FIND the most probable position of the straight line determined by the measured coordinates, each measure being equally good or of equal weight, (x_1, y_1) , (x_2, y_2) , ... (x_n, y_n) of n points, that is find a and b in the equation

$$y = ax + b. \quad (1)$$

Since $y_1 - ax_1 - b$, $y_2 - ax_2 - b$, ... $y_n - ax_n - b$ are the distances, parallel to the axis y , from the n points to the required line,

$$\left[(y_1 - ax_1 - b)^2 + (y_2 - ax_2 - b)^2 + \dots + (y_n - ax_n - b)^2 \right] \frac{1}{1-a^2} = S(d_1^2) = u, \quad (2)$$

is the sum of the squares of the normals from the n points to the required line, which sum by the principle of least squares, ANALYST, p. 183, Vol. IV, must be a minimum. Therefore

$$\frac{du}{db} = \frac{2nb - 2S(y_1) + 2aS(x_1)}{1 + a^2} = 0, \text{ or } S(y_1) - aS(x_1) - nb = 0, \quad (3)$$

where $S(y_1) = y_1 + y_2 + \dots + y_n$. And

$$\frac{du}{da} = \frac{2aS(x_1^2) - 2S(x_1y_1) + 2bS(x_1)}{1 + a^2} - \frac{[S(y_1^2) + a^2S(x_1^2) + nb^2 - 2aS(x_1y_1) - 2bS(y_1) + 2abS(x_1)]2a}{(1 + a^2)^2} = 0,$$

$$\text{or } a^2S(x_1y_1) - a^2bS(x_1) + a[S(x_1^2) - S(y_1^2)] - nab^2 + bS(x_1) + 2abS(y_1) - S(x_1y_1) = 0, \quad (4)$$

where $S(x_1y_1) = x_1y_1 + x_2y_2 + \dots + x_ny_n$.

Eliminating b from (3) and (4) and solving for a ,

$$a = \frac{S(y_1^2) - S(x_1^2) + [S(x_1)]^2 - [S(y_1)]^2}{2[S(x_1y_1) - S(x_1)S(y_1)]} \pm \sqrt{\left[1 + \left(\frac{S(y_1^2) - S(x_1^2) + [S(x_1)]^2 - [S(y_1)]^2}{2[S(x_1y_1) - S(x_1)S(y_1)]}\right)^2\right]}. \quad (5)$$

Equation (3) shows that the line passes through the centre of gravity of the n points, and therefore by (2) it must be a principal axis of them. And by (5) there are two positions of the line, at right angles to each other, which make $S(d_1^2)$ a minimum. The first value of a which gives the least minimum is the one which the problem requires.

$x_1 = 1, y_1 = 1, x_2 = 3, y_2 = 2, x_3 = 5, y_3 = 4$, gives

$$a = \frac{1 + 4 + 16 - 1 - 9 - 25 + (1 + 3 + 5)^2 - (1 + 2 + 4)^2}{2[(1 + 6 + 20) - (1 + 3 + 5)(1 + 2 + 4)]} \pm \sqrt{\left(1 + \frac{1}{16}\right)}$$

$$= -\frac{1}{4} \pm \frac{1}{4} \sqrt{17} = .78078 \text{ or } 1.2808.$$

NOTE ON THE QUANTITY g , PAGE 25, BY PROF. JOHNSON.—The value of this quantity is

$$g_n = \frac{1}{3}(2^n \pm 1);$$

for, beginning with $g_0 = 0$, we have, from the given relation

$$g_{n-1} = 2g_n \pm 1,$$

$g_1 = 1, g_2 = 2 - 1, g_3 = 2^2 - 2 + 1$, and in general $g_n = 2^{n-1} - 2^{n-2} + 2^{n-3} - \dots$, whence

$$g_n = \frac{2^n \pm 1}{2 + 1}.$$

In the table an error occurs in the value of g_{19} , which introduces a cumulative error in the subsequent values, and another error was made in forming g_{28} ; the final value should be

$$g_{50} = 375,299,968,947,541.$$